HYDRODYNAMICS OF A FLUIDIZED BED

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Using the integral features of the system (constancy of bed resistance and solid phase mass), a system of Lagrange–Euler equations is derived by a variational method and a qualitative analysis of the features of the hydrodynamics of a gaseous fluidized system is conducted.

Because of the wide use of fluidized systems in chemical technology, it is important to know the hydrodynamic laws which determine, to a considerable extent, the processes taking place in the bed. A special feature of the gaseous fluidized bed is the nonuniformity of the volumetric particle concentration and the outburst of gas bubbles through the bed, a feature which makes for poor contact with the solid. A good deal of information has accumulated as to the nature of motion of the bubbles [1, 2, 10], and they have been treated in the framework of the two-phase-system model [1, 10]. There have appeared simultaneously several papers in which [3-6, 9] statistical methods are used to investigate the hydrodynamics of a fluidized system; this requires detailed knowledge of the statistical characteristics of the ensemble under examination.

We have attempted to use the variational approach to describe the hydrodynamics of a fluidized system, based on a knowledge of its integral characteristics (bed resistance independent of gas filtration speed over a wide range, mass of solid particles in the bed volume constant).

We consider a transition process in a fluidized bed. The speed at the inlet is increased steadily from its initial value (fluidize start speed) to some ambient value (below the speed solid particles are carried away from the bed).

Within the bed we single out an element dV, appreciably larger than the volume of one particle, and introduce average values of the speed of motion of the solid and gas phases, w_i and v_i, respectively. The energy lost by the gas in filtering through the bed of particles during the transition process time (t_2-t_1) can be described by the following integral:

$$I = \int_{t_1}^{t_2} \int_{V(t)} -\left[(1-\varepsilon) \frac{d}{dt} \left(\frac{\rho_g v_i^2}{2} + \alpha p \right) \right] dV dt, \tag{1}$$

and we ignore the potential energy of the gas in the field of gravity as being very small.

Assuming that such a picture of the motion holds in the two-phase system (gas plus solid particles), we find that the energy expended by the gas as it filters through the system will be a minimum. This condition is expressed mathematically by equating the variation of integral (1) to zero, i.e.,

$$\delta \int_{t_1}^{t_2} \int_{V(t)} -\left[(1-\varepsilon) \frac{d}{dt} \left(\frac{\rho_g v_i^2}{2} + \alpha p \right) \right] dV dt = 0. \tag{2}$$

If Eq. (2) is to describe the processes occurring in the fludized bed, the special physical features of the system must be taken into account. These will be isoperimetric conditions for the problem in seeking a minimum of the functional (1).

One basic feature of the system is that the mass of solid particles in the bed is constant, i.e., no particles are carried out of the bed by the gas stream. This condition can be written as:

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$$\int_{t_1}^{t_2}\int_{V(t)} \varepsilon dV dt = C_1. \quad (3)$$

We obtain a second isoperimetric condition from the following considerations. We write the power balance for the bed as

$$\int_{V(t)} (1-\varepsilon)\frac{d}{dt}\left(\frac{\rho_g v_i^2}{2}+\alpha p\right) dV = -\int_{V(t)} \varepsilon \frac{d}{dt}\left(\frac{\rho_p w_i^2}{2}\right) dV - \int_{V(t)} \varepsilon \frac{d}{dt}(\rho_p g z)\, dV - \int_{V(t)} T dV. \quad (4)$$

Equation (4) indicates that the energy expended by the gas in the bed goes to increase the speed of the solid particles, to expansion of the bed (i.e., increase in the potential energy of particles in the field of gravity), and to dissipation. These are the terms on the right side of Eq. (4).

For the special case of transition conditions (steady fluidization), Eq. (4) will take the form

$$\int_V (1-\varepsilon)\frac{d}{dt}\left(\frac{\rho_g v_i^2}{2}+\alpha p\right) dV = -\int_V T dV, \quad (5)$$

since in that case the potential energy of the solid material as a whole and its kinetic energy of motion remain constant, i.e.,

$$\int_V \varepsilon \frac{d}{dt}\left(\frac{\rho_p w_i^2}{2}\right) dV = \int_V \varepsilon \frac{d}{dt}(\rho_p g z)\, dV = 0. \quad (6)$$

It follows from Eq. (5) that the steady case differs from the unsteady (transition) process in that the gas energy goes only to dissipation. This manifests itself experimentally as resistance of the bed to the passage of gas through it.

In analogy with electrical resistance, the resistance of the bed to gas flow is given in steady-state conditions by

$$R_{st} = \frac{\int_V T dV}{v_0}. \quad (7)$$

For the unsteady case the total bed resistance is

$$R_{unst} = \frac{\int_{V(t)} \varepsilon \frac{d}{dt}\left(\frac{\rho_p w_i^2}{2}\right) dV}{v_0(t)} + \frac{\int_{V(t)} \varepsilon \frac{d}{dt}(\rho_p g z)\, dV}{v_0(t)} + R_T, \quad (8)$$

where $R_T = \int_{V(t)} T dV / v_0(t)$.

We assume that the transition process is rather slow (i.e., quasisteady) and

$$R_{st} = R_T. \quad (9)$$

Equation (9) means that the bed resistance to gas flow associated with dissipation in unsteady conditions is equal to the bed resistance under steady conditions. It is known [7] that the resistance of a fluidized bed is independent of the gas filtration speed, which means that R_T is constant and a characteristic of the system.

From Eqs. (4) and (9), following time-integration (during the transition process), we obtain a second isoperimetric condition of the variation problem (2):

$$-\int_{t_1}^{t_2}\int_{V(t)} \left[\frac{(1-\varepsilon)\frac{d}{dt}\left(\frac{\rho_g v_i^2}{2}+\alpha p\right)+\varepsilon\frac{d}{dt}\left(\frac{\rho_p w_i^2}{2}+\rho_p g z\right)}{v_0(t)}\right] dV dt = C_2. \quad (10)$$

Thus, the hydrodynamics of the fluidized bed is determined by solving the variational problem, i.e., finding a conditional extremum of functional (1) with the isoperimetric conditions (3) and (10). As is known

[8], this kind of problem reduces to finding a condition-free extremum of the following functional:

$$I^* = \int_{t_1}^{t_2} \int_{V(t)} (F + \lambda_1 F_1 + \lambda_2 F_2)\, dV dt, \tag{11}$$

where

$$F = -(1-\varepsilon)\frac{d}{dt}\left(\frac{\rho_g v_i^2}{2} + \alpha p\right), \quad F_1 = \varepsilon,$$

$$F_2 = -\frac{(1-\varepsilon)\frac{d}{dt}\left(\frac{\rho_g v_i^2}{2} + \alpha p\right) + \varepsilon \frac{d}{dt}\left(\frac{\rho_p w_i^2}{2} + \rho_p g z\right)}{v_0(t)},$$

i.e., to the problem

$$\delta \int_{t_1}^{t_2} \int_{V(t)} (F + \lambda_1 F_1 + \lambda_2 F_2)\, dV dt = 0. \tag{12}$$

It is known [8] that the extremals of functional (11) are determined by the Euler–Lagrange equations:

$$\sum_{k=1}^{4} \frac{\partial}{\partial x_k}\left(\frac{\partial \Phi}{\partial y_{i,k}}\right) - \frac{\partial \Phi}{\partial y_i} = 0 \qquad (i = 1, \ldots, 8), \tag{13}$$

where $\Phi = F + \lambda_1 F_1 + \lambda_2 F_2$; $y_{i,k} = \partial y_i/\partial x_k$; $x_1 = x$; $x_2 = y$; $x_3 = z$; $x_4 = t$; $y_1 = v_x$; $y_2 = v_y$; $y_3 = v_z$; $y_4 = w_x$; $y_5 = w_y$; $y_6 = w_z$; $y_7 = \varepsilon$; $y_8 = p$.

Equations (13) are a nonlinear system of differential equations in partial derivatives of first order, describing the hydrodynamics of the fluidized bed. The system has eight equations for eight unknown functions $(y_1 \ldots y_8)$, and is therefore closed.

Following a transformation of Eq. (13) we write the hydrodynamic system of equations obtained for steady conditions $((d/dt)v_0(t) = 0)$ in the following form:

$$\frac{v_k}{1-\varepsilon}\cdot\frac{\partial \varepsilon}{\partial t} + \frac{1}{1-\varepsilon} v_k v_i \frac{\partial \varepsilon}{\partial x_i} = -\frac{\partial}{\partial x_k}\left(\frac{1}{2} v_i^2\right) + v_k \frac{\partial v_i}{\partial x_i} - \frac{\alpha}{\rho_g}\cdot\frac{\partial p}{\partial x_k} \qquad (k = 1, 2, 3), \tag{14}$$

$$\frac{w_k}{\varepsilon}\cdot\frac{\partial \varepsilon}{\partial t} + \frac{1}{\varepsilon} w_k w_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_k}\left(\frac{1}{2} w_i^2\right) - w_k \frac{\partial w_i}{\partial x_i} + g\delta_{zk} \quad (k = 1, 2, 3), \tag{15}$$

$$\frac{1}{1-\varepsilon}\cdot\frac{\partial \varepsilon}{\partial t} + \frac{1}{1-\varepsilon} v_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial v_i}{\partial x_i}, \tag{16}$$

$$\left(1 + \frac{\lambda_2}{v_0}\right)\left[\rho_p L(v_i) + \alpha\frac{\partial p}{\partial t} + \alpha\frac{\partial p}{\partial x_i} v_i\right] + \lambda_1 - \frac{\lambda_2}{v_0}\left[\rho_p L(w_i) + \rho_p g w_z\right] = 0, \tag{17}$$

where

$$L(v_i) = v_i \frac{\partial v_i}{\partial t} + v_k v_i \frac{\partial v_k}{\partial x_i}; \qquad L(w_i) = w_i \frac{\partial w_i}{\partial t} + w_k w_i \frac{\partial w_k}{\partial x_i}.$$

We multiply Eq. (16) term-by-term by v_k and substitute it into Eq. (14); following some transformations we have

$$\frac{\partial}{\partial x_k}\left(\frac{1}{2} v_i^2 + \alpha\frac{p}{\rho_g}\right) = 0. \tag{18}$$

It can be seen from Eq. (18) that the total gas energy inside the bed, equal to $1/\rho_g(1-\varepsilon)$, is independent of the coordinates and is a function of time. Allowing for this, it follows from Eq. (17) that the total particle energy inside the bed $(1/2)w_i^2 + gz$, equal to $1/\rho_p\varepsilon$, is also a function only of time. Then, from Eq. (15), following transformations, we obtain finally the mass balance equation for particles

$$\frac{\partial}{\partial t}(\rho_p \varepsilon) = -\frac{\partial}{\partial x_i}(\rho_p \varepsilon w_i). \tag{19}$$

The analogous relation for the gas is Eq. (16). Equations (14) and (15) are modified momentum balance equations for the gas and particles, respectively. Equation (17) gives the energy balance in the system.

In this study we do not introduce a detailed description of the nature and the laws of interaction between the solid material and the gas stream. We replace this by an integral description of these interactions, i.e., the power dissipation (9) in the bed, which we consider as embodying the whole set of local characteristics of the interaction between the gas stream and the solid material in the fluidized bed. In this case the actual system is replaced by the following simplified model. Two mutually permeable fluids, the energy of which at any time instant does not vary throughout the space, are in motion in continuum phase space. There is an energy drain with time (dissipation) in the system, this being also uniformly distributed over the phase space continuum.

We examine the question of the possible existence of particle concentration fluctuations in the bed in the region $w_z \gg w_x, w_y$. From Eq. (15) we obtain

$$w_z \frac{\partial \varepsilon}{\partial t} + w_z^2 \frac{\partial \varepsilon}{\partial z} = \varepsilon \left[g - w_z \left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} \right) \right]. \tag{20}$$

and, from Eq. (17),

$$\left(1 + \frac{\lambda_2}{v_0} \right) \rho_g \left(v_i \frac{\partial v_i}{\partial t} + \frac{\alpha}{\rho_g} \frac{\partial p}{\partial t} \right) + \lambda_1 = \frac{\lambda_2}{v_0} \rho_p \left(w_z^2 \frac{\partial w_z}{\partial z} + g w_z \right). \tag{21}$$

Making the natural assumption that $v_z \gg v_x, v_y$, we obtain

$$v_i \frac{\partial v_i}{\partial t} \sim v_z \frac{\partial v_z}{\partial t} \sim v_z^2 \frac{\partial v_z}{\partial z}. \tag{22}$$

With the same assumption, from Eq. (16) we obtain

$$v_z^2 \frac{\partial v_z}{\partial z} \sim \frac{v_z^3}{1 - \varepsilon} \cdot \frac{\partial \varepsilon}{\partial z}. \tag{23}$$

We evaluate the term $\alpha/\rho_g \cdot \partial p/\partial t$ as follows:

$$\frac{\alpha}{\rho_g} \cdot \frac{\partial p}{\partial t} \sim \frac{\alpha}{\rho_g} v_z \frac{\partial p}{\partial z} \sim \alpha \frac{\rho_p}{\rho_g} v_z g \bar{\varepsilon}. \tag{24}$$

It can be seen from the physical meaning of the term $\lambda_1 \varepsilon$ in Eq. (11) that it describes the power going to hold particles in the bed and, therefore,

$$\lambda_1 \varepsilon \sim \rho_p \bar{v}_z g \varepsilon. \tag{25}$$

Allowing for Eqs. (22)-(25), we convert Eqs. (20) and (21) to the form:

$$w_z^2 \frac{\partial \varepsilon}{\partial z} \cong \varepsilon \left[g - w_z \left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} \right) \right], \tag{20a}$$

$$\frac{\rho_g}{\rho_p} \cdot \frac{1}{1 - \varepsilon} v_z^3 \frac{\partial \varepsilon}{\partial z} + g \left(\alpha v_z \bar{\varepsilon} + \frac{\bar{v}_z v_0}{\lambda_2 + v_0} \right) \cong \frac{\lambda_2}{\lambda_2 + v_0} \left(w_z^2 \frac{\partial w_z}{\partial z} + g w_z \right). \tag{21a}$$

We note that we can assume from the physical sense of Eq. (11) that λ_1 and λ_2 are positive coefficients.

Let w_z be such that the difference appearing in Eq. (20a) is positive; we examine it under the condition $\partial w_x/\partial x > 0$; $\partial w_y/\partial y > 0$, i.e., in a region where the particles are being accelerated horizontally. It follows from Eq. (20a) that $\partial \varepsilon/\partial z > 0$, i.e., ε increases with height. On the other hand, it is clear from Eq. (21a) that the left side of that equation increases under these conditions. It is evident from the structure of Eq. (21a) that w_z also increases. As w_z increases, $g - w_z(\partial w_x/\partial x + \partial w_y/\partial y)$ and $\partial \varepsilon/\partial z$ both change sign. The right side of Eq. (21a) will decrease with decrease of ε, and therefore w_z will also decrease until the gradient of ε along the z axis becomes positive again. And then everything is repeated.

Thus, from the above reasoning it is clear that oscillations in particle concentration can exist in the fluidized bed. We substitute into Eq. (15) the value of $\partial \varepsilon/\partial t$ from Eq. (16):

$$w_k (v_i - w_i) \frac{\partial \varepsilon}{\partial x_i} + \varepsilon \left[g \delta_{zk} + w_k \frac{\partial}{\partial x_i} (v_i - w_i) + \frac{\partial}{\partial x_k} \left(\frac{1}{2} w_i^2 \right) \right] = w_k \frac{\partial v_i}{\partial x_i}.$$

We investigate Eq. (26) in three special cases:

1) $w_z \to 0$, the particles only diverge or converge horizontally. From Eq. (26) with k = 3

$$w_z(v_i - w_i)\frac{\partial\varepsilon}{\partial x_i} \cong -\varepsilon\left[g + \frac{\partial}{\partial z}\left(\frac{1}{2}w_i^2\right)\right], \tag{26a}$$

a) if $\varepsilon \neq 0$, then $\partial\varepsilon/\partial x_i \to \infty$; b) if $\varepsilon \to 0$, $\partial\varepsilon/\partial x_i$ is finite;

2) $w_y \to 0$, there is no motion of the particles in the y direction. From Eq. (26) with k = 2

$$w_y(v_i - w_i)\frac{\partial\varepsilon}{\partial x_i} \cong -\varepsilon\left[\frac{\partial}{\partial y}\left(\frac{1}{2}w_i^2\right)\right], \tag{26b}$$

a) if $\varepsilon \neq 0$, $\partial/\partial y((1/2)w_i^2) \neq 0$, then $\partial\varepsilon/\partial x_i \to \infty$; b) if $\varepsilon \to 0$, $\partial/\partial y((1/2)w_i^2) \neq 0$, then $\partial\varepsilon/\partial x_i$ is finite;

3) $(v_i - w_i) \to 0$, the particles and the gas move with the same speed. From Eq. (26a) we have: a) $\varepsilon \to 0$, then $\partial\varepsilon/\partial x_i$ is finite; b) if $\varepsilon \neq 0$, then $\partial\varepsilon/\partial x_i \to \infty$.

Thus, we have shown qualitatively that there can be discontinuities in particle concentration in a fluidized system. The variation of the functional (12), which reflects expenditure of the gas stream energy in the transition process, describes the hydrodynamics of the fluidized bed in the approximation adopted, and elucidates its basic features (self-oscillations, discontinuities of particle concentration).

NOTATION

p	is the gas pressure;
R_{st}, R_T	are, respectively, the bed resistance under steady conditions, and that associated with dissipation in the transition process;
T	is the dissipated power per unit volume of bed;
t	is the time;
v_i, w_i	are the gas and particle velocity, respectively;
$v_0(t)$, v_0	are the gas speeds at the bed entrance under unsteady and steady conditions of fluidization, respectively;
x_i $(i = 1, \ldots, 4)$	are the coordinates x, y, z and time t, respectively;
y_i $(i = 1, 2, 3)$	are the resolved components of the velocity of the particles along the axes x, y, and z;
y_i $(i = 7, 8)$	represent ε, p;
$\alpha = \mu C_V/R$;	
ε, $\bar{\varepsilon}$	are the volume concentration of the particles and its mean value;
λ_1, λ_2	are coefficients;
μ	is the gram-molecular weight of the perfect gas;
ρ_g, ρ_p	are the gas and particle density, respectively;
v_z	is the mean gas velocity.

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